HEATING OF A MATERIAL CONTAINING INTERNAL SOURCES

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UDC 536.24.01

A study of mixed radiative and conductive heat transfer in a layer of finite thickness composed of material showing selective absorption.

Research on the heating of semitransparent materials is complicated because some of the heat is transferred by conduction and some by radiation. The heating conditions then have a considerable influence on the processes responsible for the temperature distribution.

Published studies of such heating with selectively absorbing materials relate to convective heating [1]; various assumptions have been made when external radiation fluxes have to be incorporated.

Formulation. The layer of finite thickness made of semitransparent material shows convective and radiative heating.

The layer is considered as constituting a plane-parallel plate whose outer and inner boundaries are specular reflectors. The outer boundary y = 0 is transparent to the radiation, while the inner one $y = y_\delta$ is opaque (aluminum substrate). It is also assumed that the material absorbs radiation but does not scatter it.

The temperature distribution can be determined from the following system of equations [2]:

$$\rho c \frac{\partial T}{\partial t} = \frac{\partial}{\partial y}\left(\lambda \frac{\partial T}{\partial y}\right) - \operatorname{div} q_r,$$
$$\mu \frac{\partial I_\nu}{\partial y} = k_\nu (n_\nu^2 B_\nu - I_\nu), \tag{1}$$

where

$$\operatorname{div} q_r = 2\pi \int_0^1 \int_0^\infty k_\nu \frac{\partial I_\nu}{\partial \tau} d\mu d\nu, \quad \mu = \cos\theta.$$

The boundary conditions are

$$t \geqslant 0, \quad y = 0, \quad -\lambda \frac{\partial T}{\partial y} = \left(\frac{\alpha}{c_p}\right)(h_e - h_w),$$
$$I^+_{\nu,w} = r_{\nu w} I^-_{\nu w} + n_\nu^2 (1 - r_{\nu w}) I_{\nu b}; \tag{2}$$
$$y = y_\delta, \quad \frac{\partial T}{\partial y} = 0,$$
$$I^-_{\nu\delta} = r_{\nu\delta} I^+_{\nu\delta} + (1 - r_{\nu\delta}) n_\nu^2 B_{\nu\delta}. \tag{3}$$

The initial conditions are $T(y) = T_0$.

The solution to the radiative-transfer equation [1] with these boundary conditions gives the radiative heat flux and the flux divergence:

$$\operatorname{div} q_r = 2\pi \int_0^\infty k_\nu n_\nu^2 \Big\{ 2B_\nu - \int_0^1 \Big\{ \frac{m_z}{\mu} \int_0^\tau B_\nu \Big\{ m_z^{-1} +$$

Moscow Aviation Institute. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 34, No. 6, pp. 1096-1099, June, 1978. Original article submitted May 16, 1977.

 0022-0841/78/3406-0742$07.50

$$+ r_{\nu w}\left[1 + r_{\nu\delta}\exp\left(-2\,\frac{\tau_\delta - t}{\mu}\right)\right]\exp\left(-\frac{2t}{\mu}\right) + r_{\nu\delta} \times$$

$$\times\left[1 + r_{\nu w}\exp\left(-\frac{2t}{\mu}\right)\right]\exp\left(-2\,\frac{\tau_\delta - \tau}{\mu}\right)\Big\}_3 \exp\left(-\frac{\tau - t}{\mu}\right)dt +$$

$$+ \frac{1}{\mu}\int\limits_{\tau}^{\tau_\delta} B_\nu\Big\{_4 m_z^{-1} + r_{\nu\delta}\left[1 + r_{\nu w}\exp\left(-\frac{2t}{\mu}\right)\right]\exp\left(-2\,\frac{\tau_\delta - t}{\mu}\right) +$$

$$+ r_{\nu w}\left[1 + r_{\nu\delta}\exp\left(-2\,\frac{\tau_\delta - t}{\mu}\right)\right]\exp\left(-\frac{2\tau}{\mu}\right)\Big\}_4 \exp\left(-\frac{t - \tau}{\mu}\right)dt +$$

$$+ I_{\nu b}(1 - r_{\nu w})\left[1 + r_{\nu\delta}\exp\left(-2\,\frac{\tau_\delta - \tau}{\mu}\right)\right]\exp\left(-\frac{\tau}{\mu}\right)\Big\}_2 d\mu\Big\}_1 d\nu, \tag{4}$$

where

$$m_z = \left[1 - r_{\nu w} r_{\nu\delta}\exp\left(-\frac{2\tau_\delta}{\mu}\right)\right]^{-1}.$$

Complete description requires determination of the radiation intensity for the body of gas flowing around the plate. This intensity $I_{\nu b}$ is dependent on the form of the radiating gas volume and the temperature distribution in the latter. To determine $I_{\nu b}$ we have to consider a system of thermodynamic equations that describes the state of the gas, including the emission and absorption. There are difficulties of computational type, so we envisage the case of bulk emission, where the emission intensity for the gas can be found in a fairly simple fashion [3, 5]:

$$I_{\nu b} = B_{\nu b}[1 - \exp(-k_{\nu b}\delta_b)]. \tag{5}$$

Then we solve (1) with the initial and boundary conditions and also (4) and (5) to obtain the temperature distribution in the layer at any instant.

Method. We solved (1) numerically by fitting; the equation was approximated via an inexplicit six-point scheme [4]. Certain difficulties were encountered in calculating div q_r (over computer run time).

We transform (4) in such a way that the calculation of div q_r can be linked to the fitting; for this purpose we represent the integrals with respect to t as the sum of integrals over the intervals $[t_i, t_i + \Delta t_i]$, and we assume that the integrands that are not explicitly dependent on t vary linearly within the limits of integration, while the integrals over the ranges $[0, \tau]$ and $[\tau, \tau_\delta]$ can be written as sums. Certain steps then give us

$$\operatorname{div} q_r = 2\pi\int\limits_0^\infty k_\nu n_\nu^2\left[2B_{\nu n} - \int\limits_0^1 m_z(d_{1,n} + d_{2,n} + \varphi_{3,n} + \varphi_{4,n})\,d\mu\right]d\nu,$$

where

$$d_{1,n} = A_n(\mu)\sum_{i=0}^{n-1}\varphi_{1,i}\exp\left(-\frac{t_n - t_{i+1}}{\mu}\right),$$

$$d_{2,n} = B_n(\mu)\sum_{i=n}^{m-1}\varphi_{2,i}\exp\left(-\frac{t_i - t_n}{\mu}\right),$$

$$A_n(\mu) = 1 + r_{\nu\delta}\exp\left(-2\,\frac{t_m - t_n}{\mu}\right),$$

$$B_n(\mu) = 1 + r_{\nu w}\exp\left(-\frac{2t_n}{\mu}\right),$$

$$\varphi_{1,i} = \alpha_i(1 + \gamma_i)B_{\nu,i} - \beta_i(1 - \gamma_i)(B_{\nu,i} - B_{\nu,i+1}),$$

$$\varphi_{2,i} = \alpha_i(1 + \delta_i)B_{\nu,i} - [\alpha_i - \beta_i(1 - \delta_i)](B_{\nu,i} - B_{\nu,i+1}),$$

$$\varphi_{3,n} = I_{\nu b}(1 - r_{\nu w}) A(\mu) \exp\left(-\frac{t_n}{\mu}\right),$$

$$\varphi_{4,n} = B_{\nu\delta}(1 - r_{\nu\delta}) B(\mu) \exp\left(-\frac{t_m - t_n}{\mu}\right),$$

$$\alpha_i = 1 - \exp\left(-\frac{\Delta t_i}{\mu}\right), \quad \beta_i = 1 - \frac{\alpha_i \mu}{\Delta t_i},$$

$$\gamma_i = r_{\nu w} \exp\left(-\frac{2t_i + \Delta t_i}{\mu}\right),$$

$$\delta_i = r_{\nu\delta} \exp\left(-2\frac{t_m - t_i}{\mu} + \frac{\Delta t_i}{\mu}\right),$$

$$\Delta t_i = t_{i+1} - t_i.$$

Clearly, the computation time is largest for $d_{1,n}$ and $d_{2,n}$; we express $d_{1,n+1}$ in terms of $d_{1,n}$:

$$d_{1,n+1} = A_{n+1}(\mu)\left[\varphi_{1,n} + \frac{d_{1,n}}{A_n(\mu)} \exp\left(-\frac{t_{n+1} - t_n}{\mu}\right)\right]. \tag{6}$$

Similarly,

$$d_{2,n-1} = B_{n-1}(\mu)\left[\varphi_{2,n-1} + \frac{d_{2,n}}{B_n(\mu)} \exp\left(-\frac{t_n - t_{n-1}}{\mu}\right)\right]. \tag{7}$$

We thus obtain recurrent summations that give $d_{1,n}$ and $d_{2,n}$ in a simple fashion and, correspondingly, div q_r for all the nodes in the grid. Also, (6) and (7) allow one to incorporate the determination of div q_r into the fitting.

NOTATION

ρ, density; c, heat capacity; λ, thermal conductivity; T, temperature; T_0, initial wall temperature; q_r, radiant heat flux; I_ν, radiation intensity; k_ν, absorption coefficient; τ, optical coordinate; t, time; y, coordinate in rectangular system; h, enthalpy; α/c_p, heat-transfer parameter; I_ν^+, I_ν^-, forward and backward radiation intensities; r_ν, refraction coefficient; n_ν, index of refraction; B_ν, Planck's function; ν, frequency; δ_b, effective layer thickness; θ, direction of radiation; m, number of grid nodes in thickness y_δ; i, n, node numbers. Indices: w, outer edge of plate; δ, inner edge of plate; b, gas parameter; e, outer edge of boundary layer.

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